Aspects of Constrained Long-Short Equity Portfolios

Taking off the handcuffs.

Eric H. Sorensen, Ronald Hua, and Edward Qian

Institutional investors today increasingly look for sources of consistent alpha in the quest for higher risk-adjusted returns. In the case of equity investment, this means two things: 1) Find more alpha where possible; and 2) undo the traditional coupling of alpha (pure skill) with beta (pure risk premium).

The well-accepted equity market-neutral strategy (a pure long-short portfolio) is a clear beneficiary. This strategy delivers more risk-adjusted return, all things equal, but it lacks the naturally positive long-term equity risk premium accorded to a positive beta.

A fiduciary can obviously have both—high pure alpha from a manager, plus pure beta using derivatives. Many pension fiduciaries, however, have opted for solutions somewhere between the traditional long-only strategy (beta close to 1.0) and the totally decoupled solution of pure alpha ("ported" to any desired set of beta exposures). This compromise is one reason we see large flows into enhanced index strategies—consistent alpha (although lower), with an embedded beta of 1.0.

A more recent solution is some relaxation of the long-only constraint in the traditional investment guideline. In this way the resulting portfolios can invest both long and short, and continue to manage against their respective benchmarks. We refer to these hereafter as *constrained long-short portfolios*.

A manager, for example, might buy a 125% exposure in long equity positions and sell a 25% exposure in short equity positions, with the net result of 100% long systematic risk. But the total leverage to the alpha source is 150% (125% long and 25% short). Although the constrained long-short portfolio might be suboptimal compared to the market-neutral portfolio (with derivatives),

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it offers considerable benefit over *handcuffed* long-only portfolios.

Our objective is to remove the handcuffs. When we take them off, our simulations show that we can land more punches and sustain fewer hits. Using simulation under a variety of real-world assumptions, we demonstrate the net benefits of allowing shorting and the various optimal amounts of shorting. We show how the maximum information ratio for a given level of skill is achieved as a function of the long-only benchmark, the targeted tracking error, and a variety of important cost considerations.

OFF WITH HANDCUFFS—THEN WHAT?

What are the merits of the constrained long-short portfolio? First, it delivers more alpha. Second, it simplifies the overall plan's maintenance of beta integrity consistent with policy. Consider the active manager goal—beat the market cap-weighted benchmark, subject to typical tracking error constraints.

The cap-weighted index Goliaths are heavily weighted toward a set of large-cap stocks. For example, the largest 4% of S&P 500 names constitute about 70% of the index weight; the smallest 25% represent only 4%. If the active manager's skill is equal across all cap ranges, how can the manager win? He or she can't efficiently express beliefs in specific stocks.

With notional limits (no negative weights) on many of the bad stocks, there is insufficient funding for the good ones. For example, managers can underweight the small stocks by only a few basis points when they have a negative forecast. This implies long-only managers can add real value from their views on small stocks only half of the time—when the forecast is positive.

Some ability to short in the constrained long-short portfolios mitigates this drag to varying degrees. Therefore, in theory one should expect them to deliver higher risk-adjusted return than the long-only counterparts. But, is the ratio of 125 long to 25 short optimal? On what does the ratio depend?

There are key questions underlying the answer. First, what is the benchmark and its risk budget? Second, what are the incremental costs? Third, what is the character of the skill underlying the alpha, and how strong is it?

BELOW THE BELT

Shorting simply means more power put in the punch than merely underweighting a long holding relative to a benchmark. There was a time some players considered shorting veritably unAmerican (or hitting below the belt). Today, a more relevant issue is the mechanism of shorting and its inherent cost. Standard financial theory often invokes the concept of a self-financing portfolio, but leverage is not free.

With a \$100 endowment, a long-only investor can buy \$100 worth of stocks—the leverage ratio is 1:1. With leverage of, say, 125/25, for the long side, the investor engages in transactions as follows: 1) Buying \$100 worth of stocks with personal capital; 2) borrowing \$25 cash to buy an additional \$25 worth of stock; and 3) borrowing \$25 worth of stocks and short-selling them with \$25 cash proceeds. From a practical standpoint, in the third transaction, the lender (prime broker) simultaneously lends \$25 to the investor and keeps the \$25 proceeds for the short sale as collateral for the short position.

Exhibit 1 shows the investor's balance sheet.

Although the broker pays the investor an interest rebate, it is typically lower than the financing cost of borrowing. Therefore, the interest rate spread on the \$25 is a net cost that the investor must bear. For example, a spread of 1% carries the additional cost for overall 125/25 portfolios of 0.25% or 25 basis points. Similarly, the additional cost for 150/50 portfolios would 0.5% or 50 basis points.

A summary of the profit and loss statement is shown in Exhibit 2.[1]

EXHIBIT 1

Leveraged Balance Sheet

Assets	Liabilities and Equity
Assets	**Liability**
1a) stocks \$100	2b) cash \$25
2a) stocks \$25	3b) equity \$25
3a) cash rebate \$25	**Equity**
	1b) cash \$100

EXHIBIT 2

Leveraged Profit and Loss

	1a) equity return	* \$100
+	2a) equity return	* \$25
+	3a) short rebate interest rate	* \$25
–	2a) borrowing cost	* \$25
–	2a) equity return	* \$25
–	3a) borrowing cost	* \$25
=	Net Return	

THE LEVERAGE RATIO OF UNCONSTRAINED OPTIMAL PORTFOLIOS

Although the constrained long-short portfolios remove the long-only constraint, they still constrain the beta and therefore the extent of the overall short positions. Our goal is to analyze the size of the optimal short positions (long-short ratio). Optimal may be 125/25, 150/50, or the like.

We begin by deriving the long-short ratio of an unconstrained active long-short portfolio, and then proceed to the constrained portfolios. We first assume the unconstrained active portfolio is market-neutral and dollar-neutral. From basic portfolio theory, the active weight a_i in a stock is a function of its residual forecast f_i, specific risk σ_i, and a risk aversion parameter for the portfolio λ.

Mathematically, the optimal active weight is given by $a_i = f_i / \lambda\sigma_i^2$ since residuals are uncorrelated and adjusted for market-neutral and dollar-neutral. The risk aversion parameter is inversely proportional to the targeted tracking error. For high tracking error portfolios, λ is lower, giving rise to higher optimal active weights. The converse is true of low tracking error portfolios.

Suppose the benchmark weight for the stock is b_i. The total portfolio weight in stock i is then:

$$w_i = a_i + b_i = \frac{f_i}{\lambda\sigma_i^2} + b_i \tag{1}$$

Depending on the sign of w_i, the position could be either long ($w_i > 0$) or short ($w_i < 0$). Obviously, if the forecast f_i is positive, the position is long. If the forecast f_i is negative, which is equally likely for a market-neutral signal, the position is short when

$$f_i < -b_i\lambda\sigma_i^2 \tag{2}$$

The inequality indicates that there is a higher probability of a short position if the product on the right side is small. Thus, we are likely to want a short position for a given stock: 1) The lower the forecast; 2) the lower the benchmark weight; 3) the lower the specific risk; 4) the lower the risk aversion parameter; and 5) the higher the target tracking error, all else equal.

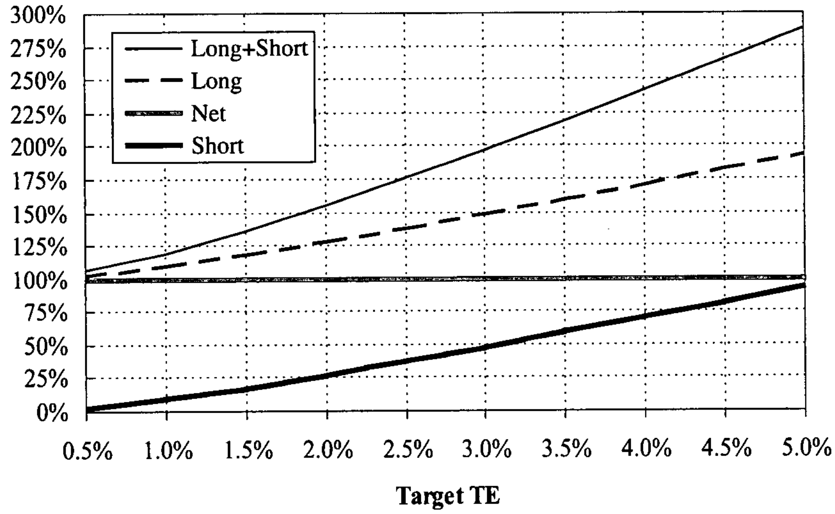
We can calculate the average size of the short position for each stock using these parameters. (The result is given in the appendix.) We then sum all the average short positions to get the total short position for the portfolio. In aggregate, the influences of these parameters carry over to the portfolio.

In summary, the total short position of an unconstrained optimal portfolio increases with lower average stock-specific risk and higher target tracking error. It turns out it also increases with benchmark concentration. For example, the total short position is lower for an equally weighted benchmark and higher for a market cap-weighted benchmark.

Exhibit 3 shows a set of long-short ratios of portfolios managed against the S&P 500 index as an increasing function of targeted tracking error. In the illustration we assume that all 500 stocks have 35% specific risk (idiosyncratic volatility). Note that the size of both long and short positions (thus leverage to the alpha source) increases with the tracking error. The net long minus short stays constant at 100%.

On the one hand, when the target tracking error is low, say, 0.5%, the total short position is small, and the portfolio is almost identical to a long-only portfolio. On the other hand, when the target tracking error rises to, say, 2%, the total short position is about 25%, and the total long position is about 125% with total leverage at 150%. When the target tracking error increases further to 3.5%, the total short position is above 50.0% and the total long position is above 150.0% with total leverage at 220.0%.

EXHIBIT 3
Optimal Long-Short Ratio of Active Portfolios for Rising Target Tracking Errors versus S&P 500



The leverage ratio of optimal portfolios also depends on the distribution of benchmark weights. Following Grinold and Kahn [2000], we model benchmark weights using a scaled lognormal distribution with a concentration index *c*. When *c* is zero, the benchmark is equally weighted, with each of the 500 stocks at a 0.20% weight. When *c* has a value of 1.2 (approximately the S&P 500), the capitalization increases from well below median to well above median for large-cap stocks.

What is the nature of the optimal long-short portfolio as the benchmark increases in concentration? Exhibit 4 shows the long-short ratio of portfolios managed against indexes with 500 stocks as the concentration rises. We perform the analysis holding the targeted tracking to 3.5%.

At the far left, *c* has a value of 0, corresponding to about 200% leverage. This is an equally weighted benchmark; an S&P 500-like benchmark would require leverage of approximately 220% to achieve 3.5% tracking error. This illustrates that benchmarks with higher concentration require more leverage. In essence, the tracking error adherence to a skewed benchmark uses up some of the leverage that would otherwise be devoted to enhancing the pure alpha.

The leverage ratio of unconstrained optimal portfolios also depends on the number of stocks in the benchmark. The Russell 2000 index, for example, is less concentrated than the S&P 500, with a concentration parameter of 0.8. With close to 2,000 stocks, its benchmark weights are uniformly lower than those in the S&P 500 index. As a result, optimal portfolios managed against the Russell 2000 generally have more short positions, and hence higher leverage, than optimal portfolios versus the S&P 500 index. This is true even though specific risks for Russell 2000 stocks are higher than those in the S&P 500.

THE INFORMATION RATIO OF LONG-SHORT PORTFOLIOS—CONSTANT IC

Unconstrained optimal portfolios have intrinsic long-short leverage ratios, which depend on both active portfolio and benchmark risk characteristics. In theory, these long-short ratios are optimal for given portfolio mandates in terms of maximizing the information ratio. If one chooses, a mandate can be implemented with pure long-short market-neutral portfolios plus a derivative for the index exposure. For the constrained long-short portfolios, however, practical constraints such as limiting individual position size and a variety of costs serve to reduce the achievable IR.

Some of the constraints are institutional. For example, prime brokers might place limits on the amount of leverage allowed in a portfolio; or for certain stocks borrowing may be difficult. These constraints would reduce the amount of shorting. Such restrictions are *discrete*, in that there are specific quantity limits.

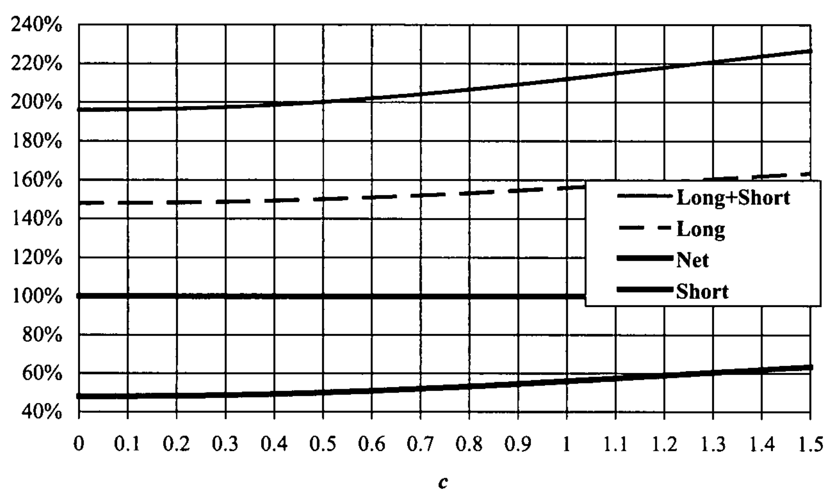
Other restrictions are cost-related, and should be thought of as *continuous*. It simply becomes more expensive as we increase shorting, and at some point it creates an excessively high drag on the net IR. Financing cost is one of these. Transaction costs also increase as the short positions rise, due to increased portfolio turnover. There may also be a need to rebalance more frequently.

In fact, shorting requires more monitoring effort in the process of trading due to the fat-tailed nature of single-stock return distributions and the potential for unrealized losses as short position prices spike from time to time. Managers must cover more names, imposing additional research work, particularly for fundamental managers.

What are the optimal solutions for a long-short ratio when we consider all these market imperfections? To understand the net benefits of constrained long-short portfo-

EXHIBIT 4

Optimal Long-Short Ratios of Active Portfolios versus Increasing Benchmark Concentrations



lios, we carry out numerical simulations: long-short portfolios with differing levels of short positions. Beginning with the limiting case of no shorting, and then relaxing the position limit on any holding, we can find the ratio that maximizes the net IR. (The appendix provides details of the simulation assumptions.) Key factors include the nature of the alpha, the tracking error target, the assumed turnover, the leveraging costs, and the trading costs.

In the simulation analyses, we first calculate the average paper excess returns from optimal portfolio weights and returns through time, and then adjust them for the return drags and frictions. In each run, we generate standardized forecasts and actual returns based on the information coefficient (IC). We then calculate the excess return of active portfolios that are managed against a benchmark with specified concentration index and a series of targeted tracking errors. These simulations are optimized with increasing relaxation of the short constraint applied to each security weight.

Exhibit 5 presents the results for simulations with a target tracking error of 3.00% and an assumed skill IC that has the expected value of 0.10 for all stocks over time. The first column describes the long-only portfolio and the last one the optimally unconstrained portfolio. This last portfolio is not totally unconstrained in that we only allow active weights as high as a reasonable 3%. As we move down the rows, we first present the average extent of optimal shorting in the aggregate portfolio (0% to 48%) and average turnover (64% to 94%).

We show the average alpha and standard deviation before considering costs associated with leverage and turnover. This produces a theoretical IR that rises from 1.59 for long-only to 2.24 for the unconstrained simulation.

Then there are costs. We estimate leverage cost and transaction cost, and subtract them from the theoretical excess return. This gives rise to a net IR calculated as the ratio of net excess return to the realized tracking error, not the target tracking error. In the case considered here, the two are indistinguishable because we assume the IC is constant.

Exhibit 5 shows that portfolio turnover increases with leverage. It averages 64% for the long-only portfolio and 94% for the unconstrained portfolio. These numbers are based on our assumption of forecast autocorrelation of 0.25.[2]

On the one hand, active short constraints have a dampening effect on portfolio turnover, as they prevent portfolios from adjusting fully to changes in forecasts. On the other hand, they have a negative impact on investment performance because of suboptimality. It is interesting to see in Exhibit 5 that turnover is basically a linear function of leverage.

EXHIBIT 5
Simulation Results for Long-Short Portfolios

Target TE = 3%

	Long-only 1	2	3	4	5	6	7	8	9	10	Unconstrained 11
Total Short	0%	9%	15%	21%	27%	33%	38%	43%	46%	47%	48%
Turnover	64%	71%	75%	79%	83%	86%	89%	91%	93%	94%	94%
Avg Alpha	4.82%	5.30%	5.56%	5.81%	6.05%	6.24%	6.41%	6.54%	6.63%	6.69%	6.72%
Std Alpha	3.04%	3.03%	3.03%	3.02%	3.01%	3.00%	3.00%	2.99%	2.99%	3.00%	3.00%
Theoretical IR	1.59	1.75	1.84	1.92	2.01	2.08	2.14	2.19	2.22	2.23	2.24
Leverage Cost	0.00%	0.09%	0.15%	0.21%	0.27%	0.33%	0.38%	0.43%	0.46%	0.47%	0.48%
Transaction Cost	0.64%	0.71%	0.75%	0.79%	0.83%	0.86%	0.89%	0.91%	0.93%	0.94%	0.94%
Net Avg Alpha	4.18%	4.50%	4.66%	4.81%	4.94%	5.05%	5.13%	5.20%	5.25%	5.28%	5.30%
Net IR	1.38	1.48	1.54	1.59	1.64	1.68	1.71	1.74	1.75	1.76	1.77
Theoretical IR decay	0.71	0.78	0.82	0.86	0.90	0.93	0.95	0.98	0.99	1.00	1.00
Net IR Decay	0.78	0.84	0.87	0.90	0.93	0.95	0.97	0.98	0.99	1.00	1.00
Transfer Coefficient	0.70	0.78	0.82	0.85	0.89	0.92	0.95	0.97	0.98	0.99	1.00

Number of stocks is 500; benchmark concentration index is 1.2; target tracking error is 3%; stock-specific risk is 35%; average IC is 0.1 with no intertemporal variation; forecast autocorrelation is 0.25; leverage cost is 1%; and transaction cost is 1%.

To calculate the net average alpha, we assume a 1% spread between long financing and short rebate and transaction cost of 1% for 100% turnover. These rates are reasonable and conservative estimates. In practice, the financing and rebate spread is subject to negotiation with prime brokers, and the transaction cost depends on many factors such as commission, bid-ask spread, and market impact. Using net average alpha, the long-only portfolio IR drops from 1.59 to 1.38, a decline of 0.21; the unconstrained optimal portfolio IR drops from 2.24 to 1.77, a much greater decline (0.47), due to higher leverage cost and transaction cost.

Finally, we compute both the theoretical and net (after-cost) IR drag, defined as ratio of IR of the constrained to the unconstrained theoretical (and net) portfolios, respectively. For example, the long-only portfolio's theoretical IR drag is 71% of unconstrained IR (1.59 versus 2.24). Its net IR drag is 78% of unconstrained net IR (1.38 versus 1.77). The constrained portfolio number 6, with an average of 33% short, achieves about 95% of unconstrained net IR (1.68 versus 1.77).

The last row in Exhibit 5 shows the transfer coefficient, defined as the correlation between the active weights in the constrained portfolios with the forecasts (see Clarke, de Silva, and Thorley [2002]). In this case, the transfer coefficients are close to theoretical IR decay but differ from the net IR decay.

Exhibit 6 graphs the theoretical IR and net IR as we relax the short constraint. Note two particular features of the graph. First, the rate of increase in IR under relaxation of the short constraints is higher in terms of theoretical IR than in terms of net IR. This is because of higher leverage cost and higher transaction cost associated with less constrained portfolios.

Second, both plots are curves, and not straight lines. The marginal increase in IR seems to be the strongest for long-only portfolios, and it diminishes as the short constraints are relaxed further.

One of many reasons for the low IR of the long-only portfolios is the inferior allocation of active risk. If a signal has uniform predictive power across stocks of all sizes, then the optimal allocation of active risk should be equal across the size spectrum. But this is not the case for the long-only portfolios, because the constraint forces more active risk be accorded to stocks with high benchmark weights.

Exhibit 7 shows the contribution to active risk from five quintiles of stocks ranked by benchmark weights (rank 1 is largest stocks and rank 5 is smallest) for the 11 portfolio simulations. The targeted tracking error is 3%. The long-only portfolio has 45% of risk in the largest quintile, and 17% in the second-largest quintile, while the remaining three quintiles each contribute roughly 13%. As we relax the short constraint, the contribution from the largest quintile declines, while the rest contribute more until we reach the unconstrained portfolio, where all quintiles contribute equal and optimal amounts—20% to active risk.

THE INFORMATION RATIO OF LONG-SHORT PORTFOLIOS—STOCHASTIC IC

One of the underlying assumptions for the simulations is a constant information coefficient. This assumption, which also underlies some results in Grinold and Kahn [2000] and Clarke, de Silva, and Thorley [2002], is often

EXHIBIT 6
Theoretical and Net IR Shown for Exhibit 5

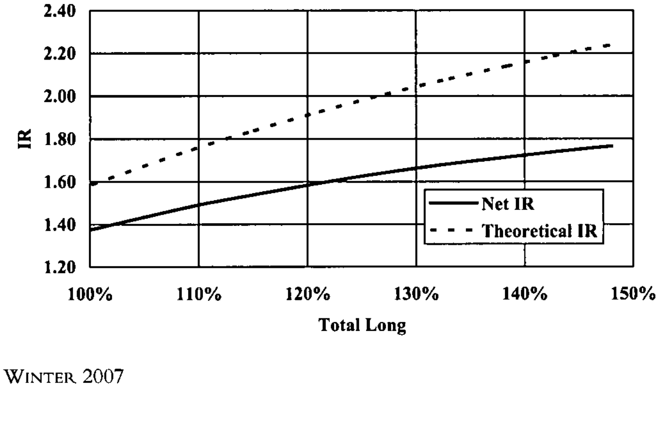
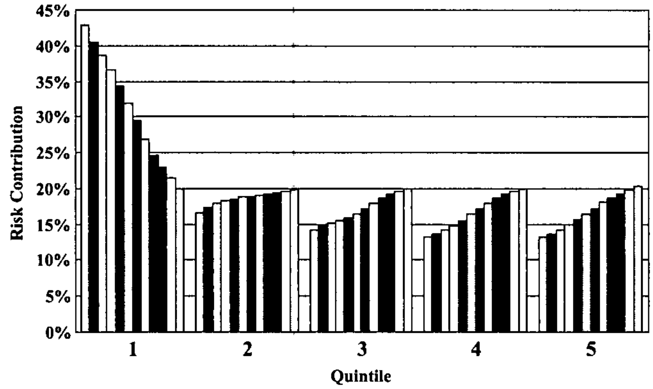


EXHIBIT 7
Risk Contributions from Quintiles of Stocks According to Size



The active risk is 3% across 500 stocks, and each quintile has 100 stocks. Each shade corresponds to portfolios with a given long-short ratio, from left to right, ranging from the long-only to unconstrained.

violated in practice. Qian and Hua [2004] show that active investment strategies bring additional risk that is not captured by generic risk models, and as a result the realized or ex post tracking error often exceeds the target or ex ante tracking error. We refer to this additional risk as *strategy risk*, and represent it by the intertemporal variation of IC. The realized tracking error is then a function of the standard deviation of IC that consists of both the intertemporal variation and the sampling error.

The IR of an active investment strategy is then given by the ratio of average IC to the standard deviation of IC:

$$IR = \frac{\overline{IC}}{\text{std}(IC)} \tag{3}$$

For example, if the intertemporal variation of IC is 0.03, the standard deviation of IC is

$$\text{std}(IC) = \sqrt{0.03^2 + \frac{1}{N}} = \sqrt{0.03^2 + \frac{1}{500}} = 0.054$$

And the IR of unconstrained portfolios is

$$IR = \frac{0.1}{0.054} = 1.86$$

Exhibit 8 shows the simulation results that take into account the additional intertemporal variation of IC, in this case, at 0.03. First notice the unconstrained portfolio (number 11) has a realized tracking error of 3.62% even though the target is 3.00%, because of additional strategy risk, and the theoretical IR is 1.86 as we indicated earlier.

Second, note that the realized tracking error for the long-only portfolio is 3.25%, closer to the target although still higher. As a result, the long-only theoretical IR is 1.46, lower than the unconstrained one by 0.42. And as we relax the no-short constraint, the realized tracking error increases.

These results indicate that more stringent range constraints have the potential benefit of controlling ex post tracking error when there is added strategy risk. To put this differently, relaxing the long-only constraint could potentially lead to higher ex post tracking error, and portfolio managers must pay extra attention to risk management.

The other characteristics of the portfolios such as percentage short and turnover rate remain roughly the same, so additional costs remain unchanged. The net IR, however, is lower in Exhibit 8 than in Exhibit 5 because of higher realized tracking error.

Finally, the differences between IR decay and transfer coefficient grow substantially greater. For instance, the transfer coefficient for the long-only portfolio is 0.70, while the net IR decay is 0.86 and the theoretical IR

EXHIBIT 8

Simulation Results for Long-Only Portfolios, Constrained Long/Short Portfolios, and Unconstrained Long-Short Portfolios

Target TE = 3%	Long-only										Unconstrained
	1	2	3	4	5	6	7	8	9	10	11
Total Short	0%	9%	15%	21%	27%	33%	38%	43%	46%	47%	48%
Turnover	64%	71%	75%	79%	83%	87%	89%	91%	93%	94%	94%
Avg Alpha	4.74%	5.23%	5.50%	5.75%	5.99%	6.21%	6.38%	6.53%	6.63%	6.70%	6.75%
Std Alpha	3.25%	3.33%	3.38%	3.42%	3.47%	3.52%	3.55%	3.58%	3.60%	3.61%	3.62%
Theoretical IR	1.46	1.57	1.63	1.68	1.73	1.76	1.80	1.82	1.84	1.86	1.86
Leverage Cost	0.00%	0.09%	0.15%	0.21%	0.27%	0.33%	0.38%	0.43%	0.46%	0.47%	0.48%
Transaction Cost	0.64%	0.71%	0.75%	0.79%	0.83%	0.86%	0.89%	0.91%	0.93%	0.94%	0.94%
Net Avg Alpha	4.10%	4.43%	4.60%	4.75%	4.89%	5.01%	5.11%	5.19%	5.25%	5.29%	5.33%
Net IR	1.26	1.33	1.36	1.39	1.41	1.42	1.44	1.45	1.46	1.47	1.47
Theoretical IR decay	0.78	0.84	0.87	0.90	0.93	0.95	0.96	0.98	0.99	1.00	1.00
Net IR Decay	0.86	0.90	0.92	0.94	0.96	0.97	0.98	0.98	0.99	1.00	1.00
Transfer Coefficient	0.70	0.78	0.82	0.85	0.89	0.92	0.95	0.97	0.98	0.99	1.00

Intertemporal variation of IC is 0.03; all other assumptions are the same as in Exhibit 5.

decay is 0.78. In general, as strategy risk grows, we find the transfer coefficient can no longer match the ratio of constrained IR to unconstrained IR.

It is worth noting that a stochastic IC assumption, which is more realistic than a constant IC assumption, makes a considerable difference in simulation results for the realized tracking error and information ratio.

SUMMARY

To relax the no-short constraint on actively managed portfolios, one should focus on the benefits and cost of constrained long-short portfolios. Relaxing the no-short constraint broadens the investment opportunities that help portfolio managers achieve higher risk-adjusted returns. At the same time, there are added costs associated with constrained long-short portfolios, including leverage cost and higher transaction costs.

We find that the benefits outweigh the costs under reasonable assumptions. In a simulation assuming active portfolio risk of 3%, an unconstrained long-short portfolio has a leverage ratio of 150/50 with a net information ratio of 1.47, while a constrained long-short portfolio with a leverage ratio of 127/27 has a net IR of 1.41. In general, higher tracking error portfolios require higher leverage ratios to achieve the same level of IR. Similarly, higher benchmark concentration leads to higher leverage ratios. Simulations for other portfolio mandates with different tracking error targets and benchmarks suggest that the optimal IR can be quite varied, depending on the mandate and the associated costs of implementation.

One important consideration for future research is the character of the alpha. What if some managers have higher information coefficients? What if their ICs are more predictive for long positions versus short positions, or vice versa? In practice, the relationship between many alpha signals and actual returns is in fact non-linear.

Our research in Sorensen, Hua, and Qian [2005] has shown that the information content of many factors and their interactions depend on context, varying across risk dimensions. In all likelihood, the contextual nature of IC affects the results for long-short portfolios.

APPENDIX

Average Short Positions

We assume the forecast follows a standard normal distribution, i.e., a normalized z score. Then both the active weight and the total weight of stock i follow a normal distribution with a standard deviation:

$$s_i = \frac{\sigma_{\text{target}}}{\sqrt{N}\sigma_i}$$

The mean of total weight is the benchmark weight b_i. The average short position size can be expressed as a conditional expectation:

$$\begin{aligned} E\left(w_i + b_i \mid w_i + b_i < 0\right) &= \frac{1}{\sqrt{2\pi}s_i}\int_{-\infty}^{-b_i}\left(x + b_i\right)\exp\left(-\frac{x^2}{2s_i^2}\right)dx \\ &= -\frac{s_i}{\sqrt{2\pi}}\exp\left(-\frac{b_i^2}{2s_i^2}\right) + b_i \cdot \mathrm{cdf}\left(-b_i, 0, s_i\right) \end{aligned}$$

The function cdf is the cumulative density function evaluated at $-b_i$ for the normal distribution with zero mean and standard deviation s_i.

Simulation of Benchmark Weights

Grinold and Kahn [2000] provide an algorithm to simulate benchmark weights based on a scaled lognormal distribution. For a given number of stocks N in the benchmark, a parameter c is used to characterize the concentration of the index. If $c = 0$, the index is equally weighted. As c increases, the index becomes more concentrated. The algorithm has four steps:

1. Discretize the probability interval (0,1) with $p_i = 1 - \frac{i - 0.5}{N}$, $i = 1, \cdots, N$.
2. Find the value of the standard normal variable that has the cumulative probability p_i i.e., $y_i = \phi^{-1}(p_i)$, where ϕ^{-1} is the inverse of the cumulative density function.
3. Transform y_i to a lognormal variable using $s_i = \exp(cy_i)$.
4. Scale s_i to obtain benchmark weight $b_i = s_i \Big/ \sum_{i=1}^{N} s_i$

Simulation Assumptions

- Investment universe and benchmark: The universe includes 500 stocks. Portfolios are managed against a 500-stock index, with the index concentration measured by the parameter c. Stock-specific risk is 35% for all stocks.

- Tracking error target: Tracking error targets range from 1% to 5%. Presented in the text is the case for 3%.
- Long-short ratio constraints: A long-short ratio constraint is imposed through a range constraint on individual stocks. Starting from long-only portfolios, which have constraints on the weights as $w_i \geq 0$, we gradually loosen the constraint to $w_i \geq -s$, where s is the short position allowed in individual stocks. For instance, if $s = 0.1\%$, we can short each stock by a maximum of 10 basis points. As s grows, the total short position grows, and the portfolio would approach the unconstrained optimal portfolio. We also set the maximum active weights at +/−3%. In most cases, this 3% limit provides solutions that are optimal for all practical examples.
- Miscellaneous portfolio constraints: In addition to targeted tracking error and range constraints on the individual stocks, another portfolio constraint is a dollar-neutral constraint—the net active weight is always zero.
- Alpha forecasts: Simulations assume normally distributed z-scores. We also assume consecutive forecasts have serial autocorrelation $\rho_f = 0.25$, which is one of factors influencing portfolio turnover. The other factors are target tracking error and leverage ratio.
- Information coefficient and returns: The risk-adjusted returns are simulated using the information coefficient (IC)—the cross-sectional correlation coefficient between the forecast and the returns. Two parameters characterize the random nature of IC: the average IC, and the standard deviation of IC. The risk-adjusted return is also assumed to be normally distributed, and its cross-sectional dispersion is unity (Qian and Hua [2004]).

ENDNOTES

[1]Jacobs and Levy [2006] depict an alternative structure set up by prime brokers that finances additional long positions with shorting. While this structure has certain tax advantages, it bears the same leverage cost.

[2]For unconstrained portfolios, the theoretical turnover is a function of stock-specific risks, target tracking error, the number of stocks, and forecast autocorrelation (Qian, Hua, and Tilney [2004]).

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